

8.7 Solving Problems with Exponential & Logarithmic Functions

Applications of Logarithmic Functions

pH VALUE

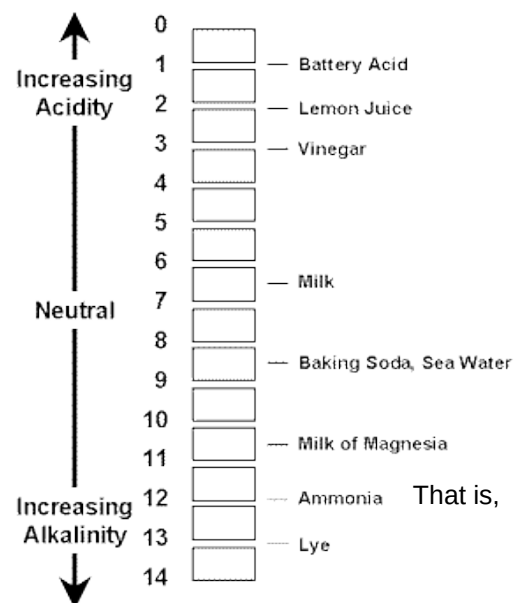
pH is the measure of acidity of a substance. The pH scale is:

The pH scale works as follows:

“A difference of one pH unit represents a tenfold (10 times) change in concentration of hydrogen ions in the solution.”

That is, each 1 unit decrease in pH represents a 10-fold increase in acidity.

For example, the pH of vinegar is 2.8. The pH of seawater is 7.8. The difference in pH is 5. So, vinegar is 10^5 times as acidic as seawater. vinegar is 100,000 times more acidic than seawater.



The relationship between pH and hydrogen ion concentration is given by the formula

$$\text{pH} = -\log[\text{H}^+]$$
 where $[\text{H}^+]$ is the concentration of hydrogen ions in mol/L.

Ex. 1 Calculate the pH if the concentration of hydrogen ions is 0.0001 mol/L.

Ex. 2 The pH of grape fruit is 3.1 whereas the pH of lemon juice is 2.6. How many times more acidic is lemon juice than a grapefruit? Round answer to 2 decimal places.

Ex. 3 The pH of black coffee is 5.1. What is the pH of a substance that is 40 times less acidic than black coffee? Round answer to 2 decimal places.

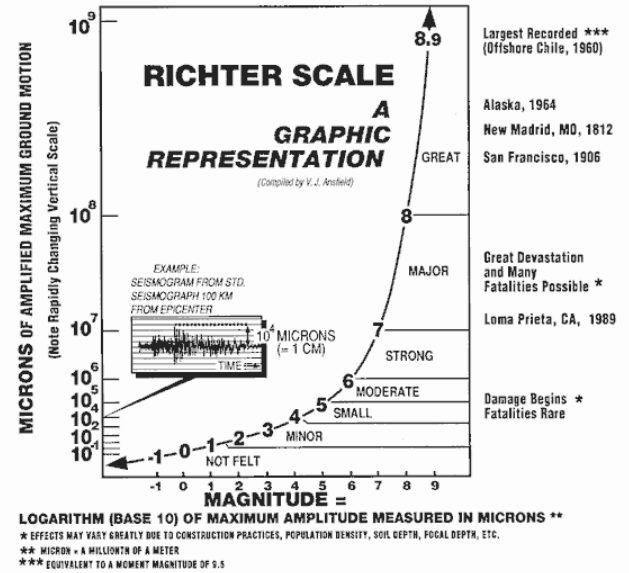
Richter Scale and Earthquakes

You have probably seen the destructive nature of earthquakes. To record an actual measurement of the energy released by an earthquake would require large numbers. For this reason, a logarithmic scale issued for comparing intensities. Charles F. Richter developed the Richter Scale, which compares the energy released at the centre of an earthquake. He defined the magnitude as:

$$M = \log \frac{I}{I_0}$$

where I is the intensity of the earthquake
 I_0 is the intensity of a reference earthquake.

He assigned the Richter numbers to earthquakes, using a reference scale.



Ex. 1 What is the magnitude of an earthquake 500 times as intense as a standard earthquake? Round answer to one decimal place.

Ex. 2 If the average earthquake measures 4.5 on the Richter Scale, how much more intense is an earthquake that measures 8? Round answer to nearest whole number.

Decibel Scale

Another application that uses logarithmic scale is sound intensity. The decibel scale provides a convenient way of comparing sound levels that range over several orders of magnitudes.

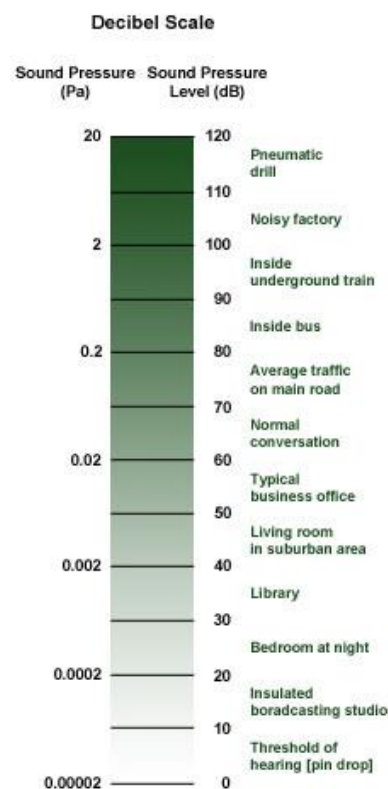
Each increase of 10 decibels (dB) represents a tenfold increase in sound intensity, or loudness.

The difference in sound levels, in decibels can be found using the equation

$$\beta_2 - \beta_1 = 10 \log \frac{I_2}{I_1}$$

where I is the intensity, measured in W/m^2
 β is the sound level, measured in dB

Ex. 1 The sound level for a whisper is 30 dB and for a normal conversation is 60 dB. How many times as intense as a whisper is the sound of a normal conversation?



Ex. 2 The sound level in normal city traffic is approximately 85 dB. The sound level while riding a snowmobile is about 32 times as intense. What is the sound level while riding a snowmobile, in decibels?

Applications with Exponential Functions (Review)

Any exponential growth or decay application can be describe using the formula

$$A = A_0(b)^{\frac{t}{k}}$$

where

A = Final amount

A_0 = original amount

b = growth/decay factor

t = elapsed time

k = time takes to decay/growth

Growth rate $r\%$	$b = 1 + r$
Decay rate $r\%$	$b = 1 - r$
Half-life	$b = \frac{1}{2}$
Doubling	$b = 2$
Tripling	$b = 3$
Compound Interest	$A = P \left(1 + \frac{i}{n}\right)^{nt}$

Ex. 1 Investment Problem

You invested \$15300 at an interest rate of 12% compounded quarterly. How long does it take for the value to triple?

Ex. 2 Exponential Growth

The population of a small town is about 2000 people in the year 1990. The growth rate is about 5% per year. How long does it take for the population to reach 10000 people?

Ex. 3 Exponential Decay

Polonium-218 is a radioactive substance that spontaneously decays in to lead-214. One minute after a 100-mg sample of polonium-218 is placed into a nuclear chamber, only 80 mg remains. Determine the half-life of polonium-218.

Extra Practice Questions for Exponential Functions Problem Solving

1. A bacteria culture starts with 3000 bacteria. After 3 h, the estimated count is 48000. What is the doubling period? [0.75h]
2. A 50-mg sample of radioactive iodine decays to 23 mg after 12 min.
 - a) Determine the half-life of the radioactive iodine. [10.7 min]
 - b) How long will it take for this sample to decay to 5 mg? [35.5 min]
3. After 90 days, a sample of silver-110, Ag^{110} has decayed to about 80% of its original size.
 - a) Determine the half life of Ag^{110} . [279.6 days]
 - b) How long will it take for this sample to decay to about 20% of its original size? [649.2 days]
4. Tenzin has purchased a minivan for \$35000. The value, V , in dollars of the minivan as a function of time, t , in years, depreciates according to the function $V(t) = 35000\left(\frac{1}{2}\right)^{\frac{t}{3}}$. How long will it take for Tenzin's minivan to depreciate to
 - a) half of its initial value? [3 years]
 - b) 15% of its initial value? [8.2 years]
5. One day, a movie trailer to the new James Bond film was posted on the Internet. Suppose that 5000 people see the video on the first day after it is posted and the number doubles every day after that.
 - a) Write an expression to describe the number of people who have seen the trailer t days after it is posted.
 - b) One week later, a second trailer is posted. Suppose that 3500 people see this trailer the first day after it is posted and that this number triples every day after that. Write an expression to describe the number of people who have seen the trailer t days after it is posted.

[Ans: a) $P(t) = 5000(2)^t$ b) $P(t) = 3500(3)^t$]